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Random-field effects in site-diluted Ising ferromagnets

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Abstract. The effects of a random field on the transition temperature in a quenched site-diluted Ising ferromagnet with coordination number $z = 3$ are investigated by the use of the effective-field theory with correlations. We find that the critical concentration of magnetic atoms at which the transition temperature reduces to zero depends on a random field. The influence of the dilution on the re-entrant phenomenon is also studied.

The random-field Ising model (RFIM) was introduced originally by Imry and Ma (1975). The interest in this model increased considerably after Fishman and Aharony (1979) have shown that the RFIM can be simulated through a diluted antiferromagnet in a uniform field. Theoretically, the RFIM has been widely investigated by the use of various techniques, such as effective-field theories (Borges and Silva 1984, Bobák *et al* 1989, Mielnicki *et al* 1989), renormalization group calculations (Aharony 1978) and Monte Carlo simulations (Landau *et al* 1978, Reed 1985).

Already early mean-field calculations (Aharony 1978) showed that for random fields with value $\pm h$ the transition becomes first order at high values of h, via a tricritical point. On the other hand, Borges and Silva (1984) have discussed the fact that the tricritical point does not exist when the coordination number z is lower than $z = 6$, by the use of an effective-field approximation based on the exact Callen (1963) single-site identity. Moreover, they have found that for an appropriate range of the random field the re-entrant phenomenon is possible in a two-dimensional RFIM. In view of the dramatic effects appearing in the RFIM, there has been an increasing stimulus in studying other systems in the presence of random fields, such as the Heisenberg model (Saxena 1981), the transverse Ising model (Saxena 1982, Sarmento and Kaneyoshi 1989) and the quenched random-bond Ising ferromagnet (Hai and Li 1989).

In this paper, using the differential operator technique introduced by Honmura and Kaneyoshi (1979), we extend the study of random-field effects to the site-diluted Ising ferromagnets. Our main aim is to examine how the critical concentration and the critical ferromagnetic frontiers are influenced by the random field. For simplicity we shall discuss only the honeycomb lattice ($z = 3$) in detail.

The Hamiltonian of the system is given by

$$H = -\tfrac{1}{2}\sum_{i,j} J_{ij}s_i s_j \xi_i \xi_j - \sum_i h_i s_i \xi_i \qquad (1)$$

where $s_i = \pm 1$, J_{ij} is the exchange interaction, ξ_i is the random variable which takes the values unity or zero, depending on whether the site i is occupied by a magnetic atom or not, and h_i is a random field, which is assumed to be randomly distributed according to the independent probability distribution function $P(h_i)$ as

$$P(h_i) = \tfrac{1}{2}[\delta(h_i - h) + \delta(h_i + h)]. \tag{2}$$

According to Callen (1963), the thermal average of $\xi_i s_i$ resulting from (1) is given by the exact formula

$$\xi_i\langle s_i\rangle = \xi_i\left\langle \tanh\left[\beta\xi_i\left(\sum_j J_{ij}\xi_j s_j + h_i\right)\right]\right\rangle \tag{3}$$

with $\beta = 1/k_B T$. Introducing the differential operator technique, we have

$$\xi_i\langle s_i\rangle = \xi_i\left\langle \prod_j [\xi_j \cosh(Dt_{ij}) + \xi_j s_j \sinh(Dt_{ij}) + 1 - \xi_j]\right\rangle \tanh(x + \beta h_i)|_{x=0} \tag{4}$$

where $D = \partial/\partial x$ is the differential operator and $t_{ij} = \beta J_{ij}$. For deriving equation (4) from equation (3), we have used both an identity

$$\exp(as_j) = \cosh a + s_j \sinh a \tag{5}$$

and the relation $\xi_i^n = \xi_i$, where n is an integer. From the decoupling approximation, namely $\langle s_j s_k \ldots s_l\rangle \simeq \langle s_j\rangle\langle s_k\rangle \ldots \langle s_l\rangle$, equation (4) may be rewritten as

$$\xi_i\langle s_i\rangle = \xi_i \prod_{j=1}^{z} [\xi_j \cosh(Dt) + \xi_j\langle s_j\rangle \sinh(Dt) + 1 - \xi_j] \tanh(x + \beta h_i)|_{x=0} \tag{6}$$

where j takes values for nearest neighbours of a site only; hence $t_{ij} = t = \beta J$. It should be noted here that this approximation is quite superior to the standard mean-field theory, since it neglects correlations between different spins but takes relations such as $\langle s_i^2\rangle = 1$ exactly into account. On the other hand, the standard mean-field theory neglects all correlations.

For a disordered system with random fields and random occupation of magnetic atoms, we must perform the random configurational average for equation (6). In the case when the random fields and random occupation of magnetic atoms are given by independent random variables, equation (6) reduces to, upon performing the random average,

$$m = c[c \cosh(Dt) + m \sinh(Dt) + 1 - c]^z f(x)|_{x=0} \tag{7}$$

where $m = \langle\xi_i\langle s_i\rangle\rangle_r$ is the averaged magnetization (Balcerzak *et al* 1985), $c = \langle\xi_i\rangle_r$ is the concentration of magnetic atoms, $\langle\ldots\rangle_r$ denotes the random average, and

$$f(x) = \tfrac{1}{2}[\tanh(x + \beta h) + \tanh(x - \beta h)]. \tag{8}$$

We are now interested in studying the transition temperature of a system. In particular, for a system with $z = 3$ (honeycomb lattice), the averaged magnetization m is given by

$$m = Am + Bm^3 \tag{9}$$

or

$$m^2 = (1 - A)/B \tag{10}$$

with

$$A = 3c \sinh(Dt)\, [c \cosh(Dt) + 1 - c]^2 f(x)|_{x=0} \tag{11}$$

$$B = c \sinh^3(Dt)\, f(x)|_{x=0}. \tag{12}$$

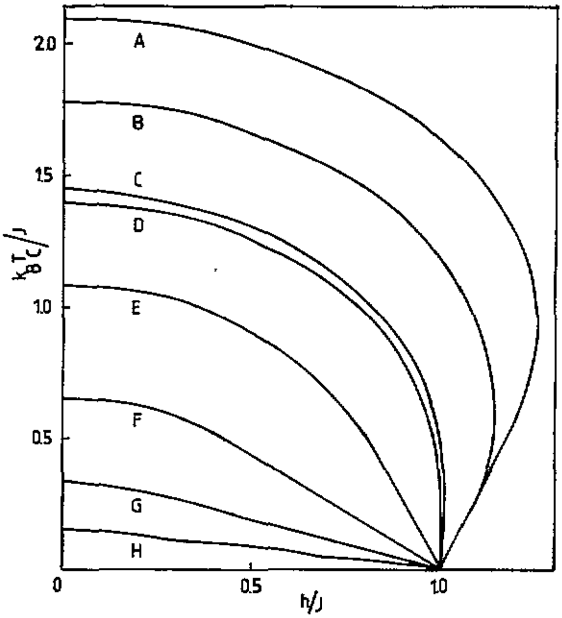

Figure 1. Phase diagrams in the (T, h)-space for different values of c: curve A, $c = 1.0$; curve B, $c = 0.9$; curve C, $c = 0.8$; curve D, $c = 0.7896$; curve E, $c = 0.7$; curve F, $c = 0.6$; curve G, $c = 0.56$; curve H, $c = 0.55751$.

The coefficients A and B can be easily calculated by the use of the mathematical relation $\exp(\lambda D) f(x) = f(x + \lambda)$. The second-order phase transition line is then determined from the relation $A = 1$.

The right-hand side of (10) must be positive. If this is not the case, the transition is of the first order, and hence the point at which $A = 1$ and $B = 0$ is the tricritical point (Bobák *et al* 1989). In the following, let us investigate the combined effect of a random-field and random-site dilution on the transition temperature, since such a problem may appear in a mixed alloy.

In figure 1, the phase diagrams of k_BT_c/J versus h/J are shown, when the concentration c is changed. As can be seen from this figure, for curve A with $c = 1$, there is rather a wide h/J-range ($1 < h/J < 1.259$) where two critical temperatures occur, which corresponds to the re-entrant phenomenon. On decrease in c this range is strongly reduced and for the critical value $c_0 = 0.7896$ (curve D) the re-entrant magnetism disappears. With further decrease in c, the ferromagnetic phase region becomes narrow and reduces to zero for another critical value $c_0 = 0.5575$. It should be noted here that our result for $c = 1$ is equivalent to that of Borges and Silva (1984) (the curve for $p = 1$ in their work). As is discussed in their work, the tricritical point does not exist for $z = 3$ in contrast with the mean-field approximation (Aharony 1978). The reason for this is that to obtain the first-order phase transition we must have at least the m^5-term in equation (9) which is possible only if $z \geqslant 6$.

In order to see the effects of a random field on the transition temperature of the site-diluted system more clearly, the critical lines in the (T, c)-space are plotted in figure 2 for selected values of h. The figure clearly expresses the fact that the T_c values as well as the critical concentrations c_0 at which T_c reduces to zero depend on the value of h. There are two critical concentrations, $c_0 = 0.5575$ and $c_0 = 0.7896$, for $0 \leqslant h/J < 1$ and $h/J = 1$, respectively. On the other hand, for $1 < h/J < 1.259$, a re-entrant phenomenon may occur. These results are consistent with those in figure 1.

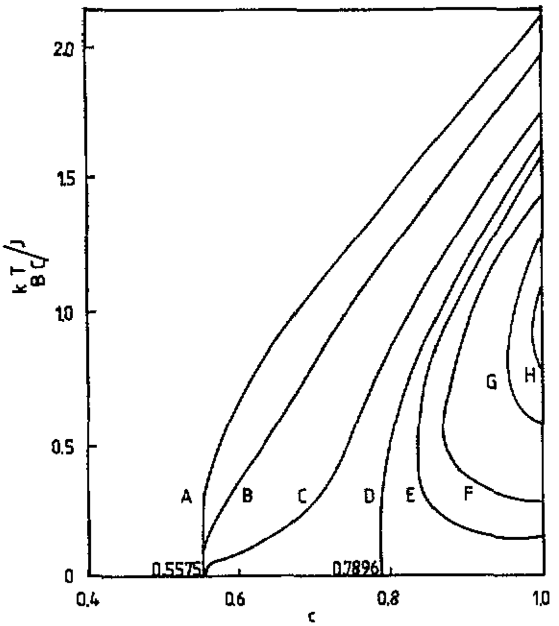

Figure 2. Phase diagrams in the (T, c)-space for different values of h: curve A, $h = 0$; curve B, $h = 0.6J$; curve C, $h = 0.9J$; curve D, $h = 1.0J$; curve E, $h = 1.05J$; curve F, $h = 1.1J$; curve G, $h = 1.2J$; curve H, $h = 1.25J$.

Let us conclude by saying that our treatment is the first attempt to study such RFIM with random-site dilution. It is difficult to obtain an experimental realization of such a system; however, a theoretical investigation for such a diluted ferromagnet will be an extension of the RFIM and may stimulate experimental investigation.

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